THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2060B Mathematical Analysis II (Spring 2017)
Tutorial 4 Correction

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One of your classmates found a logical mistake in my argument of this question. Now I attach the correction of it. Hope it helps before tomorrow's exam.

Question: Show that if f is bounded on $[a, b]$ and $g : [a, b] \to \mathbb{R}$ is another function that equals to g except on a finite set. Then g has the same upper and lower sums as f.

In particular, if two bounded functions differ only at a finite set of points, then integrability of one function implies the other and they will have the same Riemann integrals if exist.

In the tutorial, to prove

$$\overline{\int_a^b} f = \overline{\int_a^b} g,$$

I assumed WLOG that they differ only at $c \in (a, b)$, and that $f(c) < g(c)$. Hence it is trivial that $\overline{\int_a^b} f \leq \overline{\int_a^b} g$.

I said that to prove $\overline{\int_a^b} f \geq \overline{\int_a^b} g$, it suffices to show that for any $\epsilon > 0$, there is a partition P so that

$$U(f, P) - U(g, P) < \epsilon$$

But that was not enough. An exercise on supremum and infimum shows that it suffices to show that for any $\epsilon > 0$, and any partition Q, there is a partition P which is finer than Q so that

$$U(f, P) - U(g, P) < \epsilon.$$

Now since they differ at only one point, given Q, you can locate the interval(s) containing that point and note that the other sums all vanish. The remaining arguments are the same.